Honors College Algebra

Name Key
Date _____

COMPOSITION OF FUNCTIONS - DOMAIN

Find $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$ and state the domain of each composition.

1. $f(x) = \frac{3}{x-1}$ $D(-\infty, 1) \cup (1, \infty)$

$g(x) = \frac{2}{x}$ $D(-\infty, 0) \cup (0, \infty)$

$f \circ g = \frac{3}{\frac{2}{x} - 1} = \frac{3}{\frac{2-x}{x}} = \boxed{\frac{3x}{2-x}}$ D: $x \neq 0, x \neq 2$ $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$

$g \circ f = \frac{2}{\frac{3}{x-1}} = \boxed{\frac{2x-2}{3}}$ D: $(-\infty, 1) \cup (1, \infty)$

$g \circ g = \frac{2}{\frac{2}{x}} = \frac{2x}{2} = \boxed{x}$ D: $(-\infty, 0) \cup (0, \infty)$

$f \circ f = \frac{3}{\frac{3}{x-1} - 1} = \frac{3}{\frac{3-x+1}{x-1}} = \boxed{\frac{3x-3}{4-x}}$ D: $(-\infty, 1) \cup (1, 4) \cup (4, \infty)$

$x \neq 1$

$4 - x \neq 0$
$x \neq 4$

2. $f(x) = \frac{1}{x+3}$ D: $(-\infty, -3) \cup (-3, \infty)$

$g(x) = \frac{-2}{x}$ D: $(-\infty, 0) \cup (0, \infty)$

$f \circ g = \frac{1}{\frac{-2}{x} + 3} = \frac{1}{\frac{-2+3x}{x}} = \frac{x}{-2+3x}$ D: $(-\infty, 0) \cup (0, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$

$g \circ f = \frac{-2}{\frac{1}{x+3}} = -2x - 6$ D: $(-\infty, -3) \cup (-3, \infty)$

$f \circ f = \frac{1}{\frac{1}{x+3} + 3} = \frac{1}{\frac{3x+10}{x+3}} = \frac{x+3}{3x+10}$ D: $(-\infty, -\frac{10}{3}) \cup (-\frac{10}{3}, -3) \cup (-3, \infty)$

$g \circ g = \frac{-2}{\frac{-2}{x}} = x$ D: $(-\infty, 0) \cup (0, \infty)$

3. $f(x) = \frac{x}{x-1}$ D $(-\infty, 1) \cup (1, \infty)$

$g(x) = \frac{-4}{x}$ D $(-\infty, 0) \cup (0, \infty)$

$f \circ g = \frac{\frac{-4}{x}}{\frac{-4}{x} - 1} = \frac{\frac{-4}{x}}{\frac{-4-x}{x}} = \boxed{\frac{4}{4+x}}$ D: $(-\infty, -4) \cup (-4, 0) \cup (0, \infty)$

$g \circ f = \frac{-4}{\frac{x}{x-1}} = \boxed{\frac{-4x+4}{x}}$ D: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

$f \circ f = \frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} = \frac{\frac{x}{x-1}}{\frac{1}{x-1}} = \boxed{x}$ D: $(-\infty, 1) \cup (1, \infty)$

$g \circ g = \frac{-4}{\frac{-4}{x}} = \frac{-4x}{-4} = x$ D: $(-\infty, 0) \cup (0, \infty)$

4. $f(x) = \frac{x}{x+3}$ D $(-\infty, -3) \cup (-3, \infty)$

$g(x) = \frac{2}{x}$ D $(-\infty, 0) \cup (0, \infty)$

$f \circ g = \frac{\frac{2}{x}}{\frac{2}{x} + 3} = \frac{\frac{2}{x}}{\frac{2+3x}{x}} = \boxed{\frac{2}{2+3x}}$ D $(-\infty, -\frac{2}{3}) \cup (-\frac{2}{3}, 0) \cup (0, \infty)$

$g \circ f = \frac{2}{\frac{x}{x+3}} = \boxed{\frac{2x+6}{x}}$ D: $(-\infty, -3) \cup (-3, 0) \cup (0, \infty)$

$f \circ f = \frac{\frac{x}{x+3}}{\frac{x}{x+3} + 3} = \frac{\frac{x}{x+3}}{\frac{4x+9}{x+3}} = \boxed{\frac{x}{4x+9}}$ D: $(-\infty, -3) \cup (-3, -\frac{9}{4}) \cup (-\frac{9}{4}, \infty)$

$g \circ g = \frac{2}{\frac{2}{x}} = \boxed{x}$ D: $(-\infty, 0) \cup (0, \infty)$

5. $f(x) = \sqrt{x}$ D: $[0,\infty)$

$g(x) = 2x + 3$ D: $(-\infty,\infty)$

$f \circ g = \sqrt{2x+3}$ D: $[-\frac{3}{2},\infty)$

$g \circ f = 2\sqrt{x} + 3$ D: $[0,\infty)$

$f \circ f = \sqrt{\sqrt{x}} = \sqrt[4]{x}$ D: $[0,\infty)$

$g \circ g = 2(2x+3)+3 = 4x+9$ D: $(-\infty,\infty)$

6. $f(x) = \sqrt{x-2}$ D: $[2,\infty)$

$g(x) = 1 - 2x$ D: $(-\infty,\infty)$

$f \circ g = \sqrt{1-2x-2} = \sqrt{-2x-1}$ D: $(-\infty,-\frac{1}{2}]$

$g \circ f = 1 - 2\sqrt{x-2}$ D: $[2,\infty)$

$f \circ f = \sqrt{\sqrt{x-2}-2}$ D: $[6,\infty)$

$g \circ g = 1-2(1-2x) = -1+4x$ D: $(-\infty,\infty)$

7. $f(x) = x^2 + 1$ D: $(-\infty,\infty)$

$g(x) = \sqrt{x-1}$ D: $[1,\infty)$

$f \circ g = (\sqrt{x-1})^2 + 1 = x$ D: $[1,\infty)$

$g \circ f = \sqrt{x^2+1-1} = |x|$ D: $(-\infty,\infty)$

$f \circ f = (x^2+1)^2 + 1 = x^4 + 2x^2 + 2$ D: $(-\infty,\infty)$

$g \circ g = \sqrt{\sqrt{x-1}-1}$ D: $[2,\infty)$

8. $f(x) = x^2 + 4$ D: $(-\infty,\infty)$

$g(x) = \sqrt{x-2}$ D: $[2,\infty)$

$f \circ g = (\sqrt{x-2})^2 + 4 = x + 2$ D: $[2,\infty)$

$g \circ f = \sqrt{x^2+2}$ D: $(-\infty,\infty)$

$f \circ f = (x^2+4)^2 + 4 = x^4 + 8x^2 + 20$ D: $(-\infty,\infty)$

$g \circ g = \sqrt{\sqrt{x-2}-2}$ $[6,\infty)$

9. $f(x) = ax + b$ D: $(-\infty,\infty)$

$g(x) = cx + d$ R: $(-\infty,\infty)$

$f \circ g = a(cx+d)+b = acx + ad + b$ D: $(-\infty,\infty)$

$g \circ f = c(ax+b)+d = acx + bc + d$ D: $(-\infty,\infty)$

$f \circ f = a(ax+b)+b = a^2x + ab + b$ D: $(-\infty,\infty)$

$g \circ g = c(cx+d)+d = c^2x + cd + d$ D: $(-\infty,\infty)$

10. $f(x) = \frac{ax+b}{cx+d}$ $x \neq -\frac{d}{c}$ D: $(-\infty,-\frac{d}{c}) \cup (-\frac{d}{c},\infty)$

$g(x) = mx$ D: $(-\infty,\infty)$

$f \circ g = \frac{amx+b}{cmx+d}$ D: $(-\infty,-\frac{d}{cm}) \cup (-\frac{d}{cm},\infty)$

$g \circ f = \frac{max+mb}{cx+d}$ D: $(-\infty,-\frac{d}{c}) \cup (-\frac{d}{c},\infty)$

$g \circ g = m^2x$ D: $(-\infty,\infty)$

$f \circ f = \frac{a(\frac{ax+b}{cx+d})+b}{c(\frac{ax+b}{cx+d})+d} = \frac{a^2x+ab+bcx+bd}{acx+cdx+bc+d^2}$

D: $x \neq -\frac{d}{c}$, $x \neq -\frac{bc+d^2}{ac+cd}$